SIMULATION AS A TOOL TO MAKE SENSE OF THE WORLD

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An educational goal should be to prepare all students, not just those mathematically disposed, to make sound statistical inferences in different contexts. Students should be able to make decisions in the presence of uncertainty and to interpret quantitative information presented to them in the course of their professional and personal lives. This paper highlights terms such as herd immunity or efficacy that have emerged in the popular media since the onset of the COVID-19 pandemic and connects them to activities involving statistics and mathematics. Examples illustrate how simulation can be a vehicle for developing understanding of statistical concepts related to the pandemic and how simulation can engage students in reasoning from data to make sense of the world in which we live.

INTRODUCTION

"Transmission rate", "sensitivity", "efficacy", "herd immunity", "false negative", "90% effective": these terms have emerged as reporters describe the pandemic to a television audience, epidemiologists attempt to explain the virus to the public, statistical modelers predict what to expect in the future, and government officials mandate policies intended to mitigate the spread of the Covid virus. Yet, most of the public has no familiarity with these terms, and in fact evidence suggests, that those in the medical field do not always have a sound understanding of quantitative skills related to health situations (e.g., Gigerenzer & Muir Gray, 2020; Morgan, 2018). Understanding the statistics used in reference to the pandemic is critical in identifying treatments, gaining public support for intervention strategies, and enabling the public to make informed decisions related to health and to their responsibilities as citizens.

BACKGROUND

Recent documents addressing the teaching and learning of statistics emphasize the role of simulations in enabling students to make sense of statistical concepts (e.g., Guidelines for Assessment and Instruction in Statistical Education (GAISE II), 2020; International Data Science at Schools Project (IDSSP), 2018; National Council of Teachers of Mathematics, 2018). Research projects in New Zealand schools found evidence that dynamic visualizations especially designed to enhance conceptual understanding have the potential to transform the learning of statistical inference. As a result, New Zealand's introductory tertiary and secondary statistics courses focus on randomization and bootstrapping as core components of the curriculum (Pfannkuch et al., 2013). Many researchers (e,g., Cobb, 2007; delMas et al., 1999, Tintle et al., 2012) argue that using simulations to investigate concepts such as sampling distributions and confidence intervals allows students to think about the process rather than try to interpret theoretical approaches. Others suggest caution in reasoning from simulations, as in some cases, observations may be generalized incorrectly (Watkins et al., 2014). The use of dynamic interactive models is supported by research that suggests such models can be important in developing both procedural and conceptual understanding, helping students transfer mental images of concepts to visual interactive representations that lead to a better and more robust understanding of the concept (e.g., del Mas et al., 1999; Pfannkuch et al., 2020, Sacristan et al, 2010).

GAISE II (2020) suggests that simulation approaches provide a general method that applies across a wide variety of chance situations. One such situation is the statistics involved in the public discussion of the pandemic. The next section describes this opportunity to integrate statistical reasoning into the curriculum.

AN OPPORTUNITY

The mathematics and statistics students learn in early grades is visible in students' daily lives: counting, time, money, area, even fractions. The mathematics and statistics they learn in high school is more abstract and less evident in the world; applied problems are often contrived or relate to topics of little interest to students (Lovett & Lee, 2017). Probability is formulaic, and the focus in statistics is often on analysis of one variable data (Burrill, 2021). Research suggests that students are motivated to

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learn and learn more when they use real data from real contexts (Neumann et al., 2013), investigate and make sense of the world in which they live, and are engaged in something they see as useful and worthwhile (Horn, 2017; Wigfield & Eccles, 2007).

The public has been inundated with information contrasting diagnostic tests to detect COVID-19 with antibody tests to determine if someone has had COVID-19; debating which vaccine to use based on its efficacy; interpreting prediction intervals in modeling the future spread of the disease. These instances provide the education community with an opportunity to make mathematics and statistics relevant for students by investigating how such tests and vaccines work. The mathematics is not complicated: 20% of the population has contacted a disease and the screening test is 80% effective, thus out of 500 people, (0.2)(0.8)(500) or 80 people have had the disease. However, mathematical procedures obscure the presence of variability and uncertainty and suggest conclusions that seem to have one correct answer. Reporting variability is sometimes part of the discussion about the prevalence of a disease or the effectiveness of a vaccine (Tenforde et al., 2021) but not always, which leads people to question the results when one study reports 75% effectiveness and another 70%. Simulations can provide a foundation for thinking about this "noise" or variability.

The activities described below are designed to make the mathematics and statistics related to screening tests and herd immunity visible and accessible for students, using simulation to do so.

DESIGN

The activities are based on the theoretical perspective that realistic situations have a prominent role in the learning process, providing sources for initiating the development of mathematical concepts, tools, and procedures (Freudenthal,1968). Each activity was designed using five principles:

- Informal before formal

This principle is based on the philosophy that developing understanding from contextual problems can lead to informal solution strategies (Van den Heuvel-Panhuizen & Drijvers, 2020). In this approach, understanding is developed through experiences that elicit students' thinking, which is shaped by subsequent experiences to more conventional conceptual understandings in a process of progressive formalization (e.g., Gravemeijer, 2004). At this stage, students consider questions of interest: What do you know about…, read about, wonder about? They engage in a hands-on physical simulation of the situation to ground their thinking and only consider the formal mathematics at the end of the investigation.

- Delayed definitions

The second principle follows from the first where definitions and abstractions are developed through real contexts rather than pronounced, and students use relevant and useful language rather than formal terms (Rumsey, 2002). This principle is supported by research indicating curriculum materials using a delivery approach where activities are designed to be presented and learned in a logical, efficient way do not support problem solving, reasoning and communication (Choppin et al., 2020).

- Scaffolded steps to automaticity

This principle builds on Freudenthal's perspective (1971) that activity on one level is subjected to analysis on the next. This might be connected to the development of concept images, the total cognitive structure that is associated with a concept including all the mental pictures and associated properties and processes (Tall & Vinner, 1981). As students engage in new experiences related to the concept, a student's concept image changes and evolves. In this instance, the original physical model for a simulation can be gradually associated with more abstract representations. Students move from hands on simulation replicating the physical simulation to the use of technology to automate collection of data to a programed app that allows them to play with assumptions and parameters. If the connections are not clear, students will struggle when they are asked to use and interpret a computer simulation, which may become a black box.

- Reasoning from sampling distributions

Because students often struggle with the concept of distribution (Chance, et al., 2004; Wild, 2006), it is important to clearly specify the initial population, make visible the samples from that population and illustrate how repeating the simulation process leads to the distribution of a sample statistic (aka, the Simulation Process Model, Lane-Getaz, 2006). With technology, learners can repeat the sampling process many times and compare the variability both within and among samples (Chance et al., 2004;

Saldanha & Thompson, 2002). The collection of sample statistics from the simulation process can be used to create a sampling distribution of a statistic that can be analyzed using summary measures.

- Communicating and connecting

Summarizing, communicating and applying the results can help assess student understanding. Follow up activities connect the investigation to related aspects reported in the media such as positive/negative predictive values or to the mathematics involved such as an analysis of functions or the theoretical Susceptible, Infected, Recovered (SIR) model.

The following section describes two simulation-based activities related to screening test efficacy and herd immunity. The discussion of the first activity illustrates the alignment of the activity with the principles; the second is summarized briefly due to page constraints.

ACTIVITIES

Exploring Medical Test Results (2020)

“If I test positive, what is the probability I actually have the disease?” Initial discussion engages students in thinking about the information they would need to answer the question (Lane & Peres, 2006). Responses typically relate to the reliability of the test (the accuracy of the test or overall proportion of correct results), but a second factor, the prevalence of the disease, is also necessary to understand the effectiveness of a test. The activity is based on a typical flu season in the US where, depending on age and location, the prevalence rate in the winter is 20%; that is 20% of the population is likely to have the flu (Smith, 2018). Different vaccines have different accuracy rates. To begin, suppose a typical flu vaccine is 75% accurate.

- Informal before formal: Students first “act out” the situation using cards, for example the Ace, Jack, Queen, King and 10 for each of four suits in a typical card deck. An Ace represents having the flu (1 out of 5 or 20%) and a Spade represents an inaccurate test; a heart, diamond or club represent an accurate test. Students shuffle the 20 cards, draw one, and record the observation as flu-correct or incorrect test result and no flu-correct or incorrect test result. Combining class results gives a simulated set of outcomes students can use to estimate the answer to the question.
- Scaffolded steps to automaticity. Using technology to simulate the situation allows students to use larger samples with more repetitions. The first stage is to simulate the number of people with the flu in a sample of say 500 people, given that 20% of the population typically has the flu. Drawing with replacement from a jar with one blue marble and four red marbles, 20% blue (Figure 1), gave 112 out of 500 people with the flu (Figure 2). The second stage is to simulate the 75% accuracy of the test for these 112 people. One set of 112 draws with replacement from a jar with three blue marbles and one red marbles, 75% blue, gave 90/112 or about 80% with accurate tests (Figure 3), represented in the second column in Table 1. The two stages are repeated for the 388 people without the flu, producing the results in the third column in Table 1. The values in the table can be used to find the false positive rate (the probability someone tests positive but does not have the flu) and false negative rate (the probability someone tests negative but has the flu). Students should consider which of the two is least desirable in this context. Information in the table can also be used to find the conditional probability to answer the original question: given that someone tested positive, what is the probability they actually have the flu (90/103 ~ 0.47) as well as given that someone tested negative, what is the probability they do not have the flu (285/307 ~ 0.93).

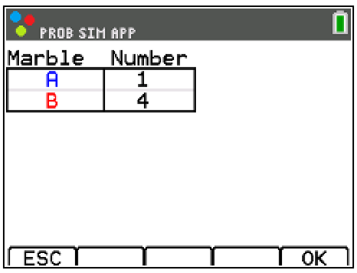


Figure 1. Setting up the model

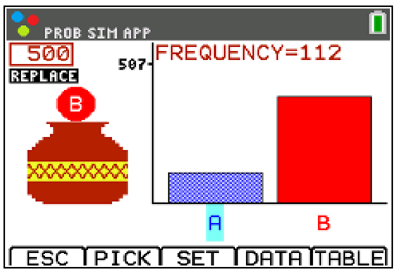


Figure 2. Simulating those with the flu

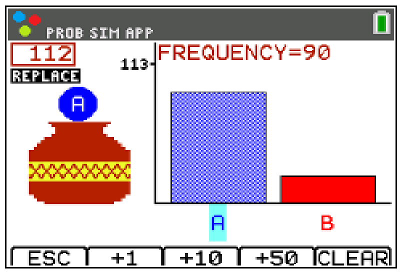
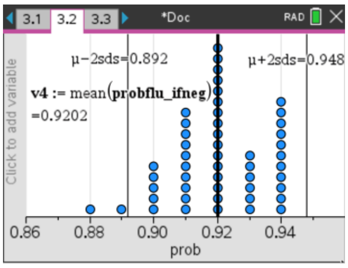


Figure 3. Simulating those testing positive with the flu

Table 1. Simulated screening test results

	Flu- infected	Not flu-infected	Total
Test positive	90	103	193
Test negative	22	285	307
Total	112	388	500

- Reasoning from sampling distributions: Combining class results produces a sampling distribution that can be analyzed to get a sense of likely answers to the original question (Figure 4). Using a simulation program that accompanies the activity, Test Simulation, students can investigate the impact of sample size (Figures 5 & 6). Changing variables (prevalence, accuracy, probabilities of true negative/positive results), allows students to explore screening tests for diseases such as measles or HIV.



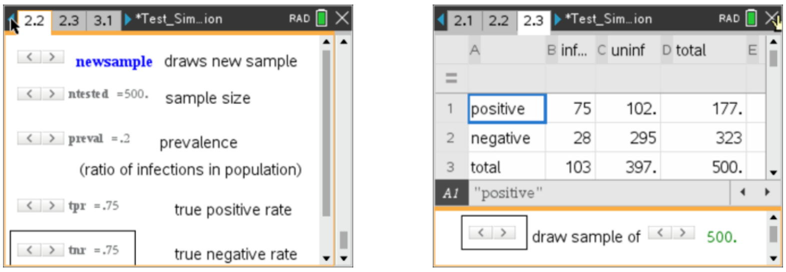


Figure 4. Probability distribution Figures 5. & 6. Simulation program allowing change of parameters

- Delayed definitions; Communicating and connecting: At the end of the activity, students should be able to describe the difference between efficacy and effectiveness, answer questions about false positives or their chances of having a disease given a test result, and understand specificity and sensitivity in the context of screening tests.

Herd Immunity (2021)

Herd immunity is another concept accessible by simulation. In this case, the variables are the transmission rate (depending on factors such as mask wearing, population density, or age), and the number of days contagious. The activity begins with a hands-on activity not given here due to page limitations. Figure 7 illustrates a graphical representation of a community of 100 people in which 10% are immune at the onset of the disease (represented by the numbers 1 to 10 and colored blue), either from vaccination or from having had the disease. Suppose a disease has a transmission rate of two (each person with the disease on average transmits the disease to two other people), and an infected person is contagious for only one day.

A random number from 11 to 100 is generated to represent a newly infected person entering the community (magenta in Figure 7). Generating two random numbers from 1 to 100 identifies the two people the newly infected person infects, and the plot is updated with two more magenta dots, using only the randomly selected people who are distinct from those originally immune. These two newly infected people each infect two others, represented by four more random numbers. This process continues until no new random numbers are generated; that is, a newly infected person only comes in contact with someone who has already been infected or was originally immune. The Herd Immunity Threshold is the sum of the immune and infected people, 70% in Figure 8. Compiling the results of many iterations gives a simulated sampling distribution for the Herd Immunity Threshold under the given conditions. An app allows students to vary sample size, the number initially immune, days contagious, and transmission rate (Figure 9) using current values for these variables, which are easily obtained from online resources related to the pandemic, to estimate the time and number of those infected before herd immunity is reached. At the end of the investigation, students analyze the theoretical SIR Model.

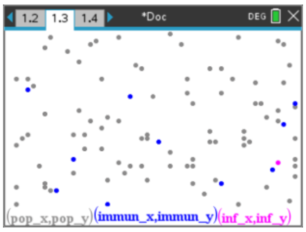


Figure 7 Sample of 100 people, 10% immune

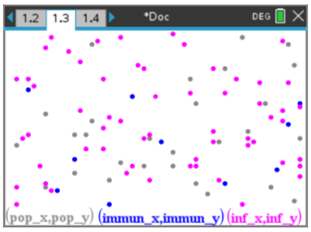


Figure 8 Herd Immunity

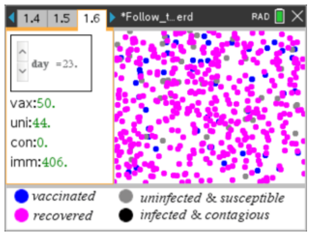


Figure 9 Changing assumptions and sample size

CONCLUSION

This paper describes activities using simulation to develop an understanding of the statistical underpinnings of concepts related to the control and spread of a disease. From the perspective that real contexts can motivate learning, a set of principles guided the development of activities that explored screening tests and herd immunity. The activities have been successfully used in 10 professional development settings and in six months were accessed by over 60 users. The responses have been very positive: “This exactly what I need to use for a project”; “Really timely”; “Great way to connect to what is going on in the world”; “These will be excellent to help students develop coding skills”.

The process might be extended to other activities where simulation can be a way to make sense of the world: the role of follow up tests; the relationship among age, deaths and hospitalization; or the effectiveness of using dogs to detect the presence of COVID. From another perspective, investigating how models predicting future spread changed over time provides an opportunity for students to understand the statistical modeling process. Collecting and analyzing data related to inequities in health systems, access to technology, or educational opportunities made visible by the pandemic could be another direction to pursue. The focus should be enabling students to interpret information related to medical issues they may encounter in their professional and personal lives.

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